A Comparative Study of Various Inclusive Indices and the Index Constructed by the Principal Components Analysis

Mishra, SK
North-Eastern Hill University, Shillong (India)

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SK Mishra
Department of Economics
North-Eastern Hill University
Shillong (India)

Introduction: A composite index (or simply an index) is constructed with an objective to obtain a synoptic or comprehensive single number, representing a vast array of measurements on the multiple aspects of a 'conceptual entity' such as general price level, cost of living, level of economic development and regional disparities (Adelman and Morris, 1967; Pal, 1975; Mishra and Chopra, 1978; Mishra and Gaikwad, 1979), quality of life (Mishra and Ngullie, 2003), human development (Cahill and Sánchez, 2001; OECD, 2003; Jahan, 2005), status of social well being (Salzman, 2003), etc. Following Saltelli (2007), Michalos et al. (2006), Nardo, *et al*. (2005), Booysen (2002) and Michalos (1980) the list of advantages of composite indices (enlisted by Michalos et al. 2006) may me presented as follows:

- A single composite index yielding a single numerical value is an excellent communications tool for use with practically any constituency, including the news media, general public, and elected and un-elected key decision-makers
- Such indices provide simple targets facilitating the focus of attention
- The simplicity of a composite index facilitates necessary negotiations about its practical value and usefulness
- Reduced transaction costs of negotiations with such indicators increase the latter's efficiency and effectiveness, probably leading to the development of better policies and programs
- Such indices provide a means to simplifying complex, multi-dimensional phenomena and measures
- They make it easier to measure and visually represent overall trends in several distinct indicators over time and/or across geographic regions
- Increase in the ease of measuring and representing trends increases our ability to predict and possibly manage future trends
- They provide a means of comparing diverse phenomena and assessing their relative importance, status or standing on the basis of some common scale of measurement, across time and space.

Such composite indices are often a weighted linear combination of a host of variables that may be symbolically expressed as **I=Xw**, where **X** is an $n \times m$ matrix of measurements in n rows (cases) and m columns (variables), **w** is a column vector of m elements and, therefore, **I** is a column vector of n index values, one for each case, summarizing all variables for the case concerned. Alternatively, it may be expressed as

$$I_i = \sum_{j=1}^{m} w_j x_{ij} = w_1 x_{i1} + w_2 x_{i2} + \ldots + w_m x_{im} \;; i = 1, 2, \ldots, n$$

There are two approaches to determining **w** or weights; first by assessing the importance of different variables with regard to the entity, idea or concept that they measure (Munda and Nardo, 2005) and secondly by obtaining those weights intrinsically. In the first case the weights are obtained from extraneous information. They might be based on the expert opinion or some other data, say, **Y**. However, in the second case, weights are derived (on some theoretical considerations - often mathematical) from the data (**X**) itself.

The Principal Components Analysis (PCA) is perhaps the most-used method to obtain weights intrinsically. The PCA determines weights of different variables such that the sum of the squared coefficients of (the product moment) correlation between the index (*I*) and the variables (**X**) is maximized. If we denote by $r(I, x_j)$ the coefficient of correlation between I and x_j, then the PCA weights are obtained such that $\sum_{j=1}^{m} r^2(I, x_j)$ is maximized.

The PCA has excellent mathematically desirable properties (Kendall and Stuart, 1968). Among those, the most important property is that the index so obtained relates to the first principal component, which explains the largest part of variation in the constituent variables (**X**). Secondly, from the residual, one may obtain the subsequent indices that are orthogonal to the first index (as well as to other indices so derived). These indices together explain the variations in the data (**X**) completely.

However, the PCA has only one objective: to construct an orthogonal index that explains the largest portion of variation in the constituent variables. This is obtained by maximizing the sum of the squared coefficient of correlation coefficients between the index (*I*) and the variables (**X**). In so doing, the PCA is not concerned with the question as to which particular variables obtain large or small weights, or which variables are only poorly represented by the index, etc. As a result what happens in practice is that when a group of variables is poorly correlated with others in the entire data set, the PCA index assigns very small loadings to some and larger loadings to the other variables if such loadings help in maximizing the sum of the squared correlation coefficients between the index (*I*) and the variables (**X**). Consequently, PCA loadings are highly *elitist* – preferring highly correlated variables to poorly correlated variables, irrespective of the (possible) contextual importance of the latter set of variables. On many occasions it is found that some (evidently) very important variables are roughly dealt with by the PCA simply because those variables exhibited widely distributed scatter or they failed to fall within a narrow band around a straight line. Further, although the PCA may permit construction of the subsequent indices (orthogonal to the first leading index), it may not be possible to use them for any comprehensive analysis since there is no dependable procedure to make an index by merging several principal component indices derived from the data (**X**).

It is not unusual, therefore, that the PCA performs poorly at constructing comprehensive indices from the variables especially if the data pertain to a system that has not evolved sufficiently (fully) such that everything determines everything else (and thus the variable drawn from the system are highly inter-correlated). It is well known that underdeveloped economies characterize underdeveloped intra-systemic interdependence as well as underdeveloped data recording mechanism. Therefore, variables drawn from an underdeveloped system exhibit widely distributed scatters and poor inter-correlations among different measurements. The elitist approach of the PCA therefore does not suit to the underdeveloped systems.

Apart from this, data may contain outliers. Those outliers may pull down (or up) correlation coefficients of the one set of variables with the rest others and thus affect the index unpredictably. The variables favored or disfavored by the PCA may obtain entirely unwarranted weights. It could have been proper in such condition to use the absolute analog of the product moment correlation coefficient suggested by Bradley (1985) who showed that if $(u_i, v_i),\ i = 1, 2, ..., n$ are n pairs of values such that the variables u and v have the same $median = 0$ and the same mean deviation (from median), $(1/n)\sum_{i=1}^{n}|u_i| = (1/n)\sum_{i=1}^{n}|v_i|$, both of which conditions may be met by any pair of variables when suitably transformed, the absolute correlation may then be defined as $\rho(u,v) = \sum_{i=1}^{n}\{|u_i + v_i| - |u_i - v_i|\} / \sum_{i=1}^{n}\{|u_i| + |v_i|\}$ that lies between $[-1,\ 1]$. However, it is a mute question whether the PCA can be carried out on the inter-correlation matrices containing a row (column) of absolute correlation coefficients while the other rows (columns) contain the product moment correlation coefficients.

Alternative Methods for Deriving Objective Weights: To draw more attention to the representation of each variable correlated strongly or weakly with the other fellow variables in the data to be used for construction of an index, albeit with some compromise on the explanatory power of the index regarding variation in the data (**X**), Mishra (2007) proposed two methods. The first, (I_1), maximizes the sum of absolute correlation coefficients of the index with the constituent variables and the second, (I_M), maximizes the minimal correlation coefficient of the index with the constituent variables. It was shown that I_1 improves the representation of weakly correlated variables while I_M has a tendency to be almost equally correlated with most of the constituent variables in which the weakly correlated variables have a secure representation. These two types of index were called 'inclusive' and 'egalitarian' respectively. It was also shown that the inclusive index (I_1) has an explanatory power only slightly less than the PCA index (I_2), which is clearly due to a trade-off between individual representation and overall representation or explanatory power. However, the egalitarian index (I_M) has much larger trade-off.

The Objectives of this Paper: In this paper we further inquire if inclusive indices (I_1) improve the representation of weakly correlated variables while the egalitarian indices (I_M) are almost equally correlated with most of the constituent variables. We also introduce a new index that, in some sense, maximizes an analogue of entropy in the data used for constructing the index. If we define $B = \sum_{j=1}^{m}|r(I, x_j)|;\ b_j = |r(I, x_j)|/B,\ \ j = 1, 2, ..., m$ where $r(I, x_j)$ =coefficient of correlation between the index (I) and the constituent variable x_j, an index may be constructed so as to maximizes $\sum_{j=1}^{m}[b_j \ln(b_j)] + B\ln(B)$. We will call such an index I_E.

The Computational Aspects: For obtaining the PCA indices one may apply the usual procedure (available in many software packages such as STATISTICA or SPSS). The procedure runs as follows. First, the (product moment) correlation matrix, $R_{m\times m}$ is obtained from the data on $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)$. Then the largest (first) eigenvalue (λ) and the associated eigenvector (v) of R are obtained and the eigenvector is standardized such that its squared Euclidean norm is equal to the eigenvalue. This standardized eigenvector (call it v^*) is used to obtain $I_2 = X^* v^*$, where X^* is obtained from X such that each variable has zero mean and unit standard deviation. In some cases, however, the eigenvector is standardized such that its Euclidean norm is equal to unity. These alternative schemes of standardization of the eigenvector only differ as to the scale of the index (I_2), but its correlation coefficients with the constituent variables remain unaltered. If one desires, the index can have non-zero mean as well and this can be done by a suitable transformation of the zero-mean index (I_2), without any effect on its correlation structure.

Alternatively, one may maximize $\sum_{j=1}^{m} r^2(I_2, x_j);\ I_2 = Xw$ directly, without going through the matrix operations detailed out above. It requires solving the nonlinear programming problem straight away using $w = (w_1, w_2, ..., w_m)$ as the vector of decision variables. Although this direct optimization method has seldom been used to construct an index, it has flexibility and generality while the matrix method is specific but (possibly) simpler.

The flexibility of the direct optimization method is very handy when we need I_1, I_M or I_E type of index. We obtain

$I_2 = Xw$ by maximization of $\sum_{j=1}^{m} r^2(I_2, x_j)$

$I_1 = Xw$ by maximization of $\sum_{j=1}^{m} |r(I_1, x_j)|$

$I_M = Xw$ by maximization of $\min_j \left(|r(I_M, x_j)|\right)$

$I_E = Xw$ by maximization of

$$\sum_{j=1}^{m}[b_j \ln(b_j)] + B\ln(B):\ B = \sum_{j=1}^{m}|r(I, x_j)|;\ b_j = |r(I, x_j)|/B,\ \ j = 1, 2, ..., m$$

We have optimized the functions by direct non-linear programming for which we use the Differential Evolution method (Mishra, 2006-a). Particle Swarm optimization (Mishra, 2006-b) also yields equally good results (not presented here to avoid duplication).

The Main Findings: We have used four sets of eight variables to construct indices. Among these sets, the first has the variables that correlate highly among themselves (Table-I.A). The other three sets contain some variables that correlate highly with some of their fellow variables, but they correlate only poorly with the others (Tables II.A through IV.A). All the four indices are obtained for each set of data. These indices are presented in Tables-I.B, II.B, III.B and IV.B. Their correlation coefficients with the constituent variables, the SAR (sum of absolute correlation coefficients) and SSR (sum of squared correlation coefficients) are presented in Tables-I.C, II.C, III.C and IV.C. The correlation coefficients among the constituent variables, among different indices, as well as across the indices and the constituent variables are presented in Tables I.D, II.D, III.D and IV.D. Different indices for each set of data have been graphically presented in Fig.-I through Fig.-IV. All indices are ordered in conformity with the ascending order of values of I_2 (the PCA index) to facilitate comparison. Instead of presenting the indices as points associated with each case (from 1 to 24), which is more appropriate, curves have been drawn only to facilitate comprehension visually.

First, we find that when all the variables are highly correlated among themselves, I_2, I_1 and I_E are very close to each other (see Tables-I.B, I.C, I.D and the Fig.-I). However, when the set contains some poorly correlated variables, I_2 differs from I_1 (as well as I_E). Interestingly, I_1 and I_E are very close to each other irrespective of the pair-wise correlations among the constituent variables. Yet, I_E has shown a leaning towards I_2 [see Tables in II, III and IV series (B, C and D), and the Fig.-II through Fig.-IV].

Secondly, I_1 (as well as I_E) has always alleviated the correlation between the index and the constituent variables that were poorly dealt with by the PCA I_2. For instance, the variables x_5 and x_6 obtained correlation coefficients –0.17 and 0.38 with I_2, which were alleviated to the values –0.37 and 0.47 by I_1 (see Table-II.D). The trade off for this was a decline in the SSR from 3.638772 (of I_2) to 3.47997 (of I_1), which is about 1.985 percent of the total variation in the data. For the third set of data the decline in explanatory power was about 2.1 percent (of the total) for alleviating the correlation (representation) of three variables, x_4, x_6, and x_8 (see Table-III.D). For the fourth set of data, correlation coefficients of five variables, x_3, x_4, x_6, x_7 and x_8 were alleviated (see Table IV.D) for a trade-off of a decline in the explanatory power by 2.178 percent. From these observations it is clear that I_1 alleviates representation of poorly correlated variables for only a small trade-off of the overall explanatory power of the index.

Thirdly, a perusal of Fig.-II through Fig.-IV reveals that alleviation of representation of weakly correlated variables by I_1 alters the rank order of cases obtained by I_2. While all indices are ordered in accordance with I_2 arranged in an ascending order (increasing monotonically), the non-monotonic movements of I_1, I_E and I_M indicate to changes in the rank order suggested by the PCA I_2. Especially, the ranking of cases by the I_M index is highly volatile.

Concluding Remarks: Construction of (composite) indices by the PCA is very common, but this method has a preference for highly correlated variables to the poorly correlated variables in the data set. Very often it fails to represent the poorly correlated variables. However, poor correlation does not entail the marginal importance, since correlation coefficients among the variables depend, apart from their linearity, also on their scatter, presence or absence of outliers, level of evolution of a system and intra-systemic integration among the different constituents of the system. Under-evolved systems often throw up the data with poorly correlated variables. If an index gives only marginal representation to the poorly correlated variables, it is elitist. The PCA index is often elitist, particularly for an under-evolved system.

In this paper we considered three alternative indices that determine weights given to different constituent variables on the principles different from the PCA. Two of the proposed indices, the one that maximizes the sum of absolute correlation coefficient of the index with the constituent variables and the other that maximizes the entropy-like function of the correlation coefficients between the index and the constituent variables are found to be very close to each other. These indices alleviate the representation of poorly correlated variables for some small reduction in the overall explanatory power (vis-à-vis the PCA index). These indices are inclusive in nature, caring for the representation of the poorly correlated variables. The third index obtained by maximization of the minimal correlation between the index and the constituent variables cares most for the least correlated variable and in so doing becomes egalitarian in nature.

It appears that neither the PCA index nor the egalitarian index can be fully justified. It is more likely that the inclusive indices (I_1 and I_E) that strike a balance between individual representation and overall representation (explanatory power) would perform better in real life. Nevertheless, it is dependent on the analyst how to choose among the different indices.

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Tables and Figures

Table-I.A: Indicator Variables for Construction of Composite Indices [Highly Correlated]								
Sl. No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	0.24746	0.62495	0.64798	0.29265	0.31671	0.45131	0.42735	0.08015
2	0.06005	0.04168	0.04671	0.08230	0.08601	0.06681	0.09165	0.09345
3	0.21551	0.22392	0.24862	0.43289	0.20651	0.19147	0.25072	0.25365
4	0.00467	0.00204	0.00207	0.00349	0.00036	0.00000	0.00060	0.00062
5	0.00492	0.00094	0.00148	0.00339	0.06350	0.04027	0.06567	0.06712
6	0.00000	0.00000	0.00000	0.00000	0.05235	0.02283	0.03290	0.03377
7	0.08357	0.05477	0.05515	0.09097	0.04490	0.02572	0.02829	0.02907
8	0.01201	0.00437	0.00596	0.01215	0.05122	0.02909	0.05154	0.05277
9	0.37148	0.48721	0.51445	0.86138	0.68763	0.92834	1.00000	1.00000
10	0.13528	0.13168	0.14674	0.25535	0.21537	0.17546	0.19945	0.20215
11	0.03036	0.02167	0.02812	0.05547	0.03834	0.04408	0.07854	0.08017
12	0.00517	0.00247	0.00273	0.00486	0.00093	0.00203	0.00179	0.00185
13	0.22977	0.23106	0.24024	0.39711	0.32794	0.40115	0.44139	0.44446
14	0.10075	0.13038	0.13673	0.22851	0.09439	0.09460	0.09475	0.09659
15	0.24962	0.29629	0.32783	0.57561	0.25105	0.24130	0.28731	0.29034
16	1.00000	1.00000	1.00000	0.99986	1.00000	1.00000	0.97507	0.31127
17	0.09500	0.09544	0.11392	0.21605	0.07342	0.14685	0.21159	0.21435
18	0.00692	0.00531	0.00670	0.01292	0.04406	0.01908	0.03115	0.03199
19	0.15371	0.23098	0.24742	0.41749	0.12368	0.12069	0.12515	0.12731
20	0.12861	0.14135	0.16284	0.29275	0.12030	0.15318	0.18549	0.18811
21	0.48198	0.64806	0.63219	0.99368	0.45382	0.62726	0.56703	0.56978
22	0.38457	0.58160	0.60495	1.00000	0.33558	0.38847	0.40284	0.40595
23	0.26555	0.46573	0.46191	0.73598	0.18568	0.29337	0.30829	0.31136
24	0.00167	0.00766	0.01020	0.02037	0.00000	0.00067	0.00000	0.00000

Table-I.B: Composite Indices Obtained by Different Methods [Ref Variables Table-I.A]									
Sl	I_2	I_M	I_1	I_E	Sl	I_2	I_M	I_1	I_E
1	1.079414	0.36413	1.056081	1.057227	13	0.937040	0.538982	0.943005	0.942854
2	0.198332	0.118026	0.199657	0.199622	14	0.330894	0.166297	0.330367	0.330430
3	0.689426	0.378483	0.691368	0.691366	15	0.853914	0.445329	0.855006	0.855063
4	0.004868	0.003617	0.004838	0.004840	16	2.564157	1.062231	2.519834	2.522082
5	0.088566	0.05599	0.090258	0.090194	17	0.394920	0.246977	0.400026	0.399850
6	0.051678	0.026676	0.052226	0.052208	18	0.056314	0.030495	0.056834	0.056818
7	0.143639	0.083649	0.142771	0.142825	19	0.517690	0.246265	0.516119	0.516253
8	0.078064	0.049483	0.079316	0.079270	20	0.465196	0.256904	0.467424	0.467383
9	2.009407	1.123417	2.024317	2.023906	21	1.694556	0.886993	1.694174	1.694394
10	0.504692	0.269166	0.506018	0.506020	22	1.380812	0.686166	1.379353	1.379588
11	0.129412	0.085831	0.131575	0.131496	23	1.014063	0.515119	1.014063	1.014188
12	0.007571	0.00498	0.007550	0.007552	24	0.012471	0.002931	0.012337	0.012345

Table-I.C: Correlation of Variables with Different Composite [Ref Variables Table-I.A & Indices I.B]										
Var	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	SAR	SSR
I_2	0.946761	0.952049	0.953948	0.929121	0.966369	0.979891	0.970493	0.788174	7.486806	7.033167
I_M	0.888074	0.888074	0.891592	0.942540	0.931946	0.966049	0.968422	0.888074	7.364771	6.788959
I_1	0.943117	0.948906	0.950980	0.931407	0.964780	0.980130	0.971582	0.796657	7.487560	7.031865
I_E	0.943282	0.949048	0.951116	0.931311	0.964854	0.980123	0.971538	0.796286	7.487558	7.031977
SAS=Sum of Absolute Correlation Coefficients; SSR=Sum of Squared Correlation Coefficients										

Table-I.D: Correlation Coefficients Among Variables and Indices [[Ref Variables Table-I.A & Indices I.B]

Variable	X1	X2	X3	X4	X5	X6	X7	X8	I2	IM	I1	IE
X1	1.00	.94	.94	.87	.95	.90	.88	.59	.95	.89	.94	.94
X2	.94	1.00	1.00	.90	.89	.90	.87	.60	.95	.89	.95	.95
X3	.94	1.00	1.00	.90	.89	.90	.88	.61	.95	.89	.95	.95
X4	.87	.90	.90	1.00	.82	.85	.84	.79	.93	.94	.93	.93
X5	.95	.89	.89	.82	1.00	.98	.97	.72	.97	.93	.96	.96
X6	.90	.90	.90	.85	.98	1.00	.99	.81	.98	.97	.98	.98
X7	.88	.87	.88	.84	.97	.99	1.00	.84	.97	.97	.97	.97
X8	.59	.60	.61	.79	.72	.81	.84	1.00	.79	.89	.80	.80
I2	.95	.95	.95	.93	.97	.98	.97	.79	1.00	.98	1.00	1.00
IM	.89	.89	.89	.94	.93	.97	.97	.89	.98	1.00	.98	.98
I1	.94	.95	.95	.93	.96	.98	.97	.80	1.00	.98	1.00	1.00
IE	.94	.95	.95	.93	.96	.98	.97	.80	1.00	.98	1.00	1.00

Off-Diagonal Entries in the Red are Significant at 5% Probability

Fig.-I. Relationship Among Different Composite Indices [Ref. Indices Table-I.B]

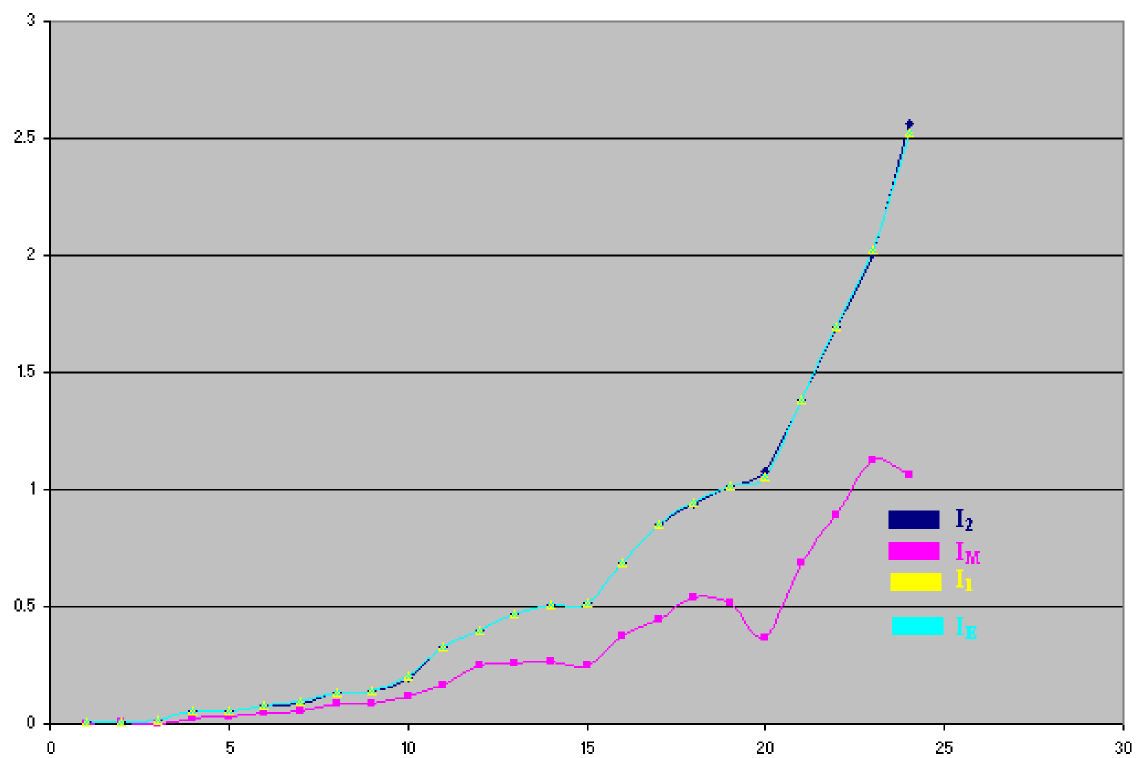


Table-II.A: Indicator Variables for Construction of Composite Indices [Poorly Correlated]								
Sl. No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	0.24746	0.62495	0.64798	0.98526	0.53466	0.39441	0.20494	0.38059
2	0.06005	0.04168	0.04671	0.81953	0.34072	0.30900	0.24717	0.42699
3	0.21551	0.22392	0.24862	0.75169	0.07558	1.00000	0.05978	0.74561
4	0.00467	0.00204	0.00207	0.62952	0.69993	0.62658	0.08976	0.00000
5	0.00492	0.00094	0.00148	0.37364	0.40331	0.46080	0.73544	0.14438
6	0.00000	0.00000	0.00000	0.00382	0.44343	0.80199	0.00000	0.37479
7	0.08357	0.05477	0.05515	0.14273	0.81714	0.51229	0.66290	0.80686
8	0.01201	0.00437	0.00596	0.00000	0.78624	0.70126	0.58950	0.33715
9	0.37148	0.48721	0.51445	0.94753	0.32142	0.83483	1.00000	1.00000
10	0.13528	0.13168	0.14674	0.75008	0.53330	0.89228	0.75648	0.29370
11	0.03036	0.02167	0.02812	0.52677	0.50320	0.27864	0.16180	0.28583
12	0.00517	0.00247	0.00273	0.81192	0.67010	0.13235	0.18545	0.20847
13	0.22977	0.23106	0.24024	1.00000	0.50611	0.43962	0.72437	0.61293
14	0.10075	0.13038	0.13673	0.07053	0.60184	0.00000	0.62472	0.34690
15	0.24962	0.29629	0.32783	0.20606	1.00000	0.48399	0.14823	0.50274
16	1.00000	1.00000	1.00000	0.84209	0.50033	0.79510	0.79321	0.19507
17	0.09500	0.09544	0.11392	0.35220	0.52443	0.29656	0.35003	0.25322
18	0.00692	0.00531	0.00670	0.79915	0.57212	0.51521	0.25717	0.11219
19	0.15371	0.23098	0.24742	0.21522	0.60198	0.81727	0.56037	0.64080
20	0.12861	0.14135	0.16284	0.45753	0.00000	0.80053	0.72405	0.61814
21	0.48198	0.64806	0.63219	0.74621	0.40213	0.62240	0.95548	0.50585
22	0.38457	0.58160	0.60495	0.63480	0.60728	0.31909	0.90734	0.95641
23	0.26555	0.46573	0.46191	0.35542	0.84484	0.91816	0.29377	0.41605
24	0.00167	0.00766	0.01020	0.14215	0.25842	0.18922	0.61566	0.49687

Table-II.B: Composite Indices Obtained by Different Methods [Ref Variables Table-II.A]									
Sl	I_2	I_M	I_1	I_E	Sl	I_2	I_M	I_1	I_E
1	1.125449	0.691817	0.940740	0.956974	13	0.928561	0.945244	0.929377	0.936279
2	0.437819	0.605365	0.486051	0.487176	14	0.377971	0.117180	0.223698	0.235215
3	0.881326	1.161851	1.066787	1.062896	15	0.633218	0.160483	0.363657	0.382202
4	0.210942	0.300350	0.139033	0.144432	16	2.060263	1.067789	1.727033	1.755478
5	0.348284	0.531365	0.376710	0.378368	17	0.380738	0.293795	0.297690	0.305053
6	0.195389	0.364572	0.229844	0.229819	18	0.311787	0.475802	0.296464	0.299828
7	0.499766	0.496664	0.448307	0.456081	19	0.747547	0.725261	0.725769	0.732526
8	0.291834	0.341532	0.227550	0.233963	20	0.793596	1.142705	1.004999	0.999831
9	1.505166	1.513450	1.589971	1.596008	21	1.485616	1.098683	1.376886	1.39183
10	0.745028	0.946549	0.788813	0.791959	22	1.375272	0.931124	1.233855	1.250889
11	0.263581	0.300388	0.223115	0.227638	23	0.933953	0.580389	0.741194	0.757039
12	0.232466	0.236987	0.139987	0.147184	24	0.319021	0.447995	0.372676	0.373255

Table-II.C: Correlation of Variables with Different Composite [Ref Variables Table-II.A & Indices II.B]										
Var	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	SAR	SSR
I_2	0.936702	0.951937	0.953816	0.511988	-0.166793	0.379617	0.566275	0.436675	4.903802	3.638772
I_M	0.584170	0.577289	0.585303	0.577289	-0.577289	0.577289	0.577289	0.577289	4.633205	2.683408
I_1	0.847254	0.852676	0.857515	0.547537	-0.372181	0.465816	0.600943	0.532267	5.076190	3.479970
I_E	0.853871	0.859913	0.864598	0.546234	-0.360623	0.461316	0.600651	0.528412	5.075618	3.497309
SAS=Sum of Absolute Correlation Coefficients; SSR=Sum of Squared Correlation Coefficients										

Table-II.D: Correlation Coefficients Among Variables and Indices [[Ref Variables Table-II.A & Indices II.B]

Variable	X1	X2	X3	X4	X5	X6	X7	X8	I2	IM	I1	IE
X1	1.00	.94	.94	.40	-.04	.31	.45	.23	.94	.58	.85	.85
X2	.94	1.00	1.00	.43	.01	.27	.41	.29	.95	.58	.85	.86
X3	.94	1.00	1.00	.43	.00	.27	.40	.31	.95	.59	.86	.86
X4	.40	.43	.43	1.00	-.30	.08	.11	.04	.51	.58	.55	.55
X5	-.04	.01	.00	-.30	1.00	-.17	-.18	-.21	-.17	-.58	-.37	-.36
X6	.31	.27	.27	.08	-.17	1.00	.07	.20	.38	.58	.47	.46
X7	.45	.41	.40	.11	-.18	.07	1.00	.43	.57	.58	.60	.60
X8	.23	.29	.31	.04	-.21	.20	.43	1.00	.44	.58	.53	.53
I2	.94	.95	.95	.51	-.17	.38	.57	.44	1.00	.78	.97	.97
IM	.58	.58	.59	.58	-.58	.58	.58	.58	.78	1.00	.91	.91
I1	.85	.85	.86	.55	-.37	.47	.60	.53	.97	.91	1.00	1.00
IE	.85	.86	.86	.55	-.36	.46	.60	.53	.97	.91	1.00	1.00

Off-Diagonal Entries in the Red are Significant at 5% Probability

Fig.-II. Relationship Among Different Composite Indices [Ref. Indices Table-II.B]

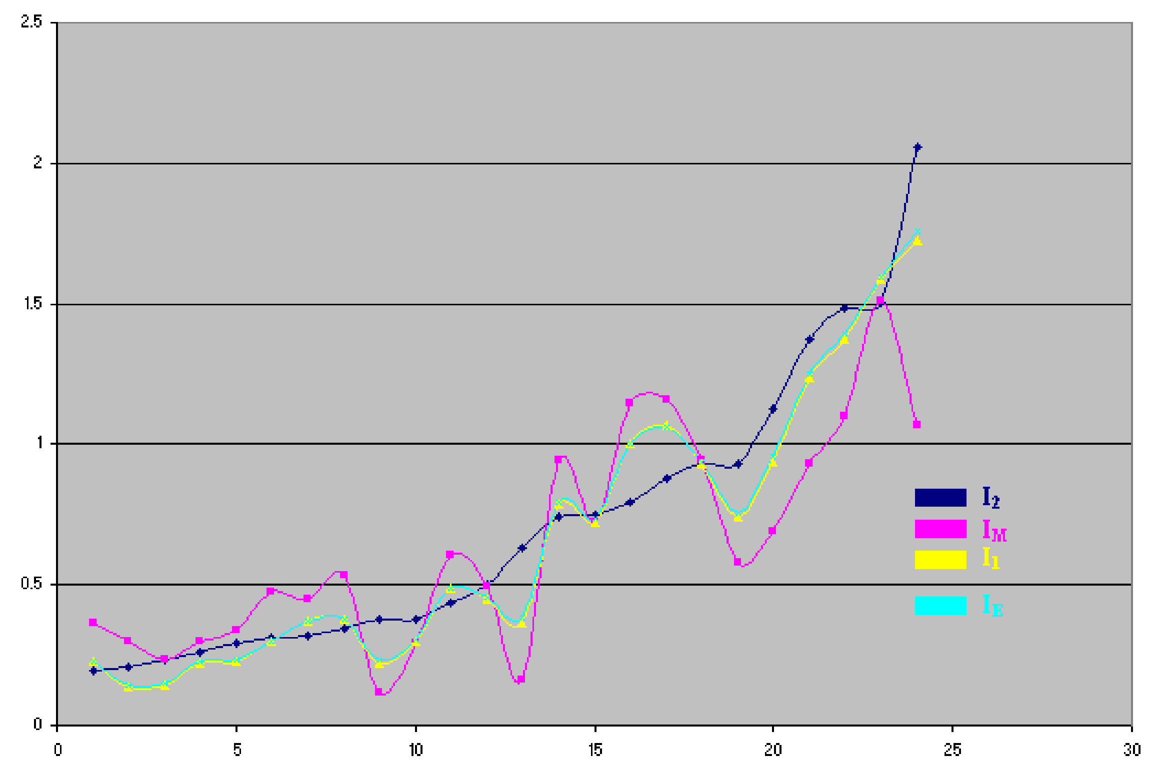


Table-III.A: Indicator Variables for Construction of Composite Indices [Poorly Correlated]								
Sl. No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	0.24746	0.62495	0.64530	1.00000	0.48439	0.25953	0.10990	0.43327
2	0.06005	0.04168	0.36793	0.90801	0.37204	0.33344	0.29963	0.48453
3	0.21551	0.22392	0.12246	0.67552	0.04525	1.00000	0.00000	0.80680
4	0.00467	0.00204	0.38290	0.73010	0.78395	0.69234	0.13950	0.00000
5	0.00492	0.00094	0.07724	0.44322	0.44683	0.50271	0.92201	0.14866
6	0.00000	0.00000	0.09666	0.03003	0.49347	0.86528	0.01387	0.44619
7	0.08357	0.05477	0.74887	0.14521	0.88956	0.56247	0.84635	0.97911
8	0.01201	0.00437	0.00000	0.02033	0.85454	0.75777	0.74490	0.39172
9	0.37148	0.48721	0.89033	0.70394	0.11269	0.51601	0.87274	0.79682
10	0.13528	0.13168	0.32328	0.75292	0.52323	0.89464	0.89386	0.27378
11	0.03036	0.02167	0.75736	0.59169	0.56191	0.31146	0.19800	0.31684
12	0.00517	0.00247	0.39756	0.93403	0.75234	0.17698	0.25891	0.25563
13	0.22977	0.23106	0.92212	0.96992	0.44989	0.32784	0.75502	0.56130
14	0.10075	0.13038	0.44711	0.00289	0.64341	0.00000	0.77143	0.38477
15	0.24962	0.29629	0.97997	0.00000	1.00000	0.44172	0.09592	0.49252
16	1.00000	1.00000	0.51977	0.52392	0.17676	0.44431	0.62379	0.10544
17	0.09500	0.09544	0.38364	0.32430	0.57030	0.28660	0.37963	0.21877
18	0.00692	0.00531	0.51589	0.91611	0.63214	0.56833	0.33681	0.12430
19	0.15371	0.23098	0.63596	0.08082	0.63192	0.83975	0.67841	0.73261
20	0.12861	0.14135	1.00000	0.40818	0.00000	0.80856	0.85891	0.67849
21	0.48198	0.64806	0.23095	0.41917	0.29050	0.42237	0.99341	0.37558
22	0.38457	0.58160	0.82704	0.29142	0.55329	0.20774	1.00000	1.00000
23	0.26555	0.46573	0.68763	0.09593	0.86276	0.87165	0.26973	0.37692
24	0.00167	0.00766	0.05871	0.17606	0.31965	0.23676	0.79869	0.61079

Table-III.B: Composite Indices Obtained by Different Methods [Ref Variables Table-III.A]									
Sl	I_2	I_M	I_1	I_E	Sl	I_2	I_M	I_1	I_E
1	0.651733	0.818771	0.748653	0.743082	13	0.657447	1.036658	0.896935	0.884985
2	0.213099	0.728379	0.424731	0.409833	14	0.285439	0.478063	0.362768	0.359784
3	0.385989	0.514145	0.325942	0.328320	15	0.337320	0.272618	0.213000	0.221695
4	-0.155489	0.136302	-0.166052	-0.170653	16	1.409851	0.770389	1.118269	1.142971
5	0.069696	0.349755	0.126488	0.121414	17	0.141003	0.325231	0.168031	0.165503
6	-0.121018	-0.076426	-0.288529	-0.281552	18	0.018980	0.420868	0.134952	0.124445
7	0.324372	0.697248	0.409374	0.405274	19	0.401334	0.400253	0.305872	0.313528
8	-0.104151	0.107688	-0.218741	-0.215419	20	0.675407	0.638039	0.747407	0.749557
9	1.032543	1.064830	1.143588	1.143399	21	0.896517	0.736519	0.808321	0.817525
10	0.261768	0.488108	0.266733	0.264948	22	1.016361	1.109601	1.107143	1.107846
11	0.151442	0.460668	0.285860	0.277170	23	0.380115	0.123095	0.117860	0.134196
12	0.001039	0.624263	0.224154	0.206151	24	0.196108	0.553662	0.322833	0.315328

Table-III.C: Correlation of Variables with Different Composite [Ref Variables Table-III.A & Indices III.B]										
Var	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	SAR	SSR
I_2	0.892224	0.894489	0.481349	0.103277	-0.523490	-0.169397	0.423706	0.354061	3.841993	2.446160
I_M	0.468925	0.49504	0.468925	0.468925	-0.468930	-0.468930	0.468925	0.468925	3.7775160	1.7843010
I_1	0.719992	0.736127	0.542095	0.296654	-0.557102	-0.360363	0.474812	0.412594	4.099740	2.278047
I_E	0.733698	0.748858	0.541317	0.282511	-0.557073	-0.348860	0.474634	0.411634	4.098585	2.298691
SAS=Sum of Absolute Correlation Coefficients; SSR=Sum of Squared Correlation Coefficients										

Table-III.D: Correlation Coefficients Among Variables and Indices [[Ref Variables Table-III.A & Indices III.B]

Variable	X1	X2	X3	X4	X5	X6	X7	X8	I2	IM	I1	IE
X1	1.00	.94	.27	.03	-.37	-.07	.22	.05	.89	.47	.72	.73
X2	.94	1.00	.32	.05	-.31	-.11	.17	.12	.89	.50	.74	.75
X3	.27	.32	1.00	.09	.06	-.16	.07	.37	.48	.47	.54	.54
X4	.03	.05	.09	1.00	-.33	-.15	-.19	-.25	.10	.47	.30	.28
X5	-.37	-.31	.06	-.33	1.00	-.08	-.23	-.19	-.52	-.47	-.56	-.56
X6	-.07	-.11	-.16	-.15	-.08	1.00	-.15	.07	-.17	-.47	-.36	-.35
X7	.22	.17	.07	-.19	-.23	-.15	1.00	.30	.42	.47	.47	.47
X8	.05	.12	.37	-.25	-.19	.07	.30	1.00	.35	.47	.41	.41
I2	.89	.89	.48	.10	-.52	-.17	.42	.35	1.00	.75	.94	.95
IM	.47	.50	.47	.47	-.47	-.47	.47	.47	.75	1.00	.92	.91
I1	.72	.74	.54	.30	-.56	-.36	.47	.41	.94	.92	1.00	1.00
IE	.73	.75	.54	.28	-.56	-.35	.47	.41	.95	.91	1.00	1.00

Off-Diagonal Entries in the Red are Significant at 5% Probability

Fig.-III. Relationship Among Different Composite Indices [Ref. Indices Table-III.B]

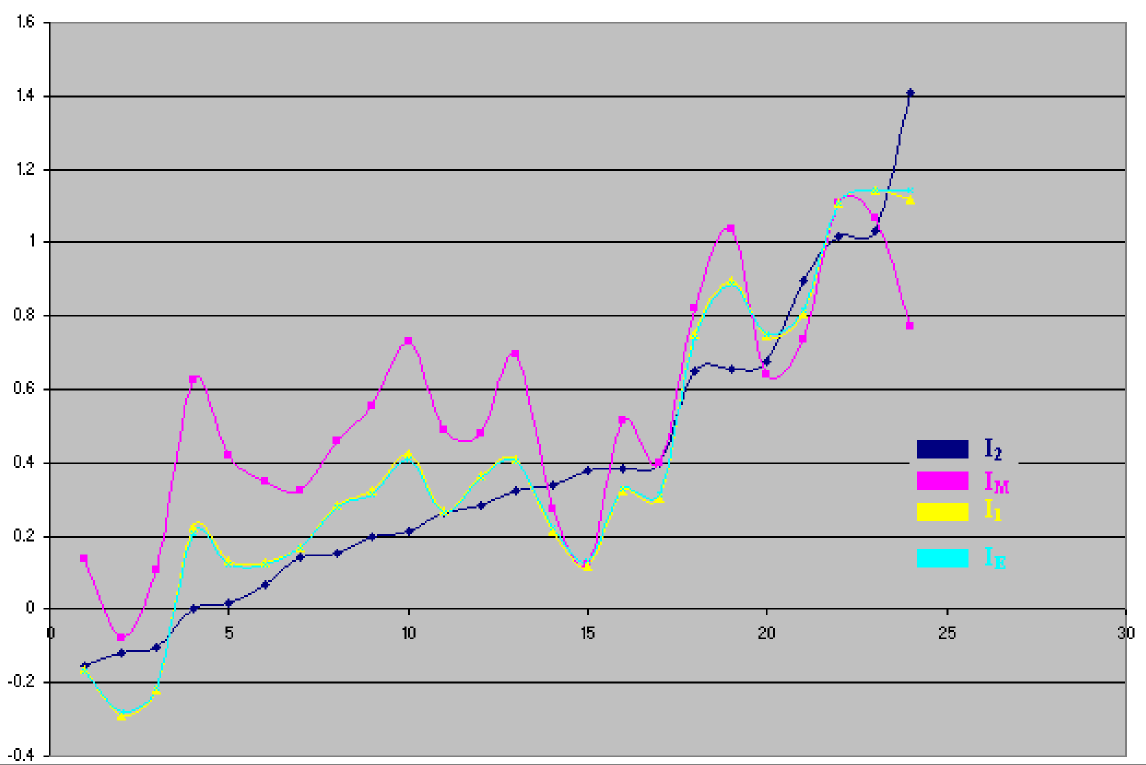


Table-IV.A: Indicator Variables for Construction of Composite Indices [Poorly Correlated]								
Sl. No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	0.24746	0.62495	0.59656	1.00000	0.47371	0.23122	0.09670	0.43659
2	0.06005	0.04168	0.36946	0.92859	0.37870	0.33857	0.31651	0.48790
3	0.21551	0.22392	0.10233	0.67130	0.03881	1.00000	0.00000	0.80370
4	0.00467	0.00204	0.38867	0.76192	0.80181	0.70615	0.16189	0.00000
5	0.00492	0.00094	0.07874	0.48210	0.45608	0.51151	0.94886	0.14633
6	0.00000	0.00000	0.09857	0.07938	0.50411	0.87856	0.03202	0.45392
7	0.08357	0.05477	0.75502	0.18382	0.90495	0.57300	0.87533	1.00000
8	0.01201	0.00437	0.00000	0.06887	0.86906	0.76964	0.77069	0.39647
9	0.37148	0.48721	0.85708	0.66173	0.06831	0.44908	0.82186	0.72743
10	0.13528	0.13168	0.31517	0.76225	0.52109	0.89514	0.90931	0.26249
11	0.03036	0.02167	0.76607	0.62239	0.57439	0.31835	0.21469	0.31737
12	0.00517	0.00247	0.40348	0.96070	0.76983	0.18635	0.28270	0.26163
13	0.22977	0.23106	0.91405	0.96157	0.43794	0.30438	0.74877	0.53544
14	0.10075	0.13038	0.44165	0.03303	0.65224	0.00000	0.79399	0.38547
15	0.24962	0.29629	0.96482	0.00000	1.00000	0.43284	0.09410	0.47866
16	1.00000	1.00000	0.43752	0.47409	0.10797	0.37067	0.57185	0.08047
17	0.09500	0.09544	0.37933	0.34760	0.58006	0.28451	0.38762	0.20508
18	0.00692	0.00531	0.52312	0.94252	0.64490	0.57948	0.35916	0.12449
19	0.15371	0.23098	0.62319	0.09259	0.63828	0.84447	0.69729	0.73891
20	0.12861	0.14135	1.00000	0.42275	0.00000	0.81025	0.87508	0.67812
21	0.48198	0.64806	0.17777	0.37246	0.26676	0.38038	0.97977	0.33419
22	0.38457	0.58160	0.78475	0.24730	0.54181	0.18436	1.00000	0.98803
23	0.26555	0.46573	0.65625	0.07961	0.86657	0.86189	0.26836	0.35843
24	0.00167	0.00766	0.05916	0.22005	0.33267	0.24674	0.82941	0.62545

Table-IV.B: Composite Indices Obtained by Different Methods [Ref Variables Table-IV.A]									
Sl	I_2	I_M	I_1	I_E	Sl	I_2	I_M	I_1	I_E
1	0.531098	0.925353	0.182593	0.221711	13	0.494170	1.150057	0.356003	0.389267
2	0.094656	0.841262	-0.091308	-0.063412	14	0.213905	0.552373	0.368402	0.376231
3	0.244917	0.652166	-0.058258	-0.022551	15	0.219757	0.380701	0.229124	0.246618
4	-0.269473	0.306820	-0.582179	-0.552165	16	1.335452	0.876993	0.845077	0.892770
5	-0.027031	0.490731	-0.127556	-0.105971	17	0.056142	0.418350	-0.019365	-0.002165
6	-0.213773	0.074337	-0.307416	-0.290635	18	-0.110342	0.580703	-0.384740	-0.353482
7	0.159879	0.865343	0.346443	0.364903	19	0.247875	0.563812	0.275847	0.298739
8	-0.208298	0.278327	-0.226687	-0.207978	20	0.498274	0.793722	0.525584	0.549162
9	0.861887	1.136477	0.753595	0.787038	21	0.794450	0.813497	0.580709	0.614520
10	0.109549	0.674369	-0.154601	-0.117721	22	0.866519	1.180156	0.965953	0.991286
11	0.036302	0.576362	-0.048515	-0.028578	23	0.245266	0.269341	0.075130	0.103762
12	-0.104548	0.741485	-0.305911	-0.279128	24	0.111973	0.650391	0.223784	0.235157

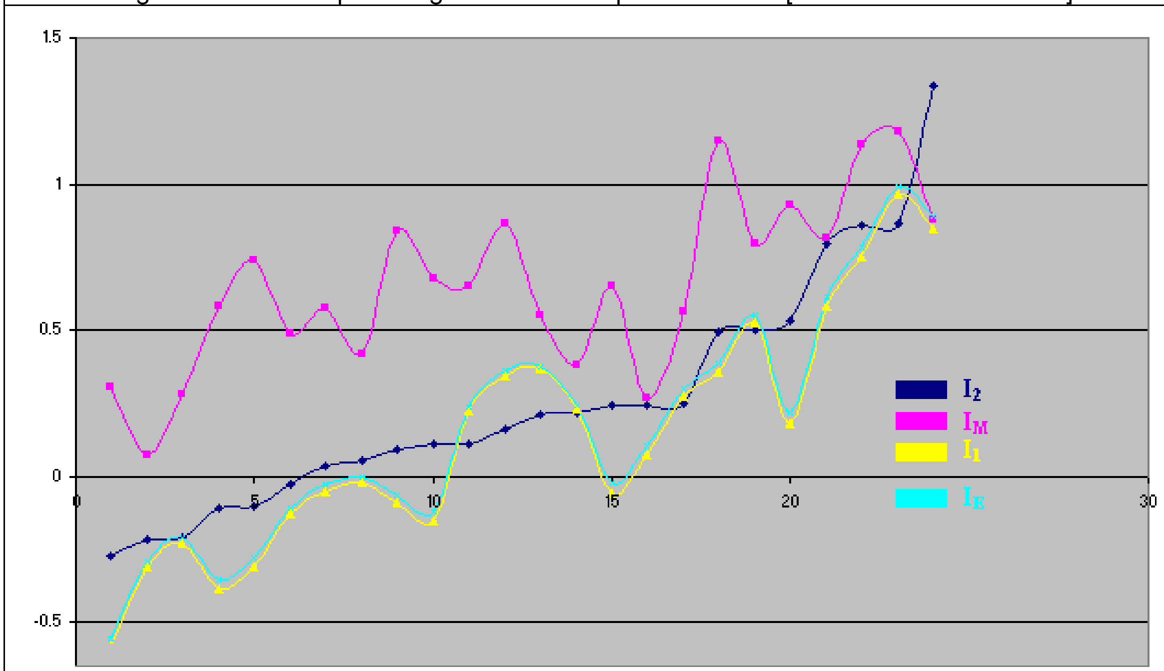
Table-IV.C: Correlation of Variables with Different Composite [Ref Variables Table-IV.A & Indices IV.B]										
Var	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	SAR	SSR
I_2	0.909969	0.907878	0.379255	0.007804	-0.566301	-0.280218	0.359750	0.266566	3.677741	2.395877
I_M	0.448620	0.469218	0.448620	0.448620	-0.484044	-0.448620	0.448620	0.448620	3.644981	1.662023
I_1	0.725619	0.732870	0.466649	-0.265822	-0.432589	-0.350864	0.542376	0.514956	4.031744	2.221633
I_E	0.737322	0.744495	0.466621	-0.250255	-0.440816	-0.345379	0.537675	0.508116	4.030679	2.239160
SAS=Sum of Absolute Correlation Coefficients; SSR=Sum of Squared Correlation Coefficients										

Table-IV.D: Correlation Coefficients Among Variables and Indices [[Ref Variables Table-IV.A & Indices IV.B]

Variable	X1	X2	X3	X4	X5	X6	X7	X8	I2	IM	I1	IE
X1	1.00	.94	.21	-.04	-.42	-.15	.16	.01	.91	.45	.73	.74
X2	.94	1.00	.25	-.03	-.36	-.18	.12	.07	.91	.47	.73	.74
X3	.21	.25	1.00	.06	.07	-.17	.04	.35	.38	.45	.47	.47
X4	-.04	-.03	.06	1.00	-.30	-.15	-.22	-.29	.01	.45	-.27	-.25
X5	-.42	-.36	.07	-.30	1.00	-.03	-.21	-.16	-.57	-.48	-.43	-.44
X6	-.15	-.18	-.17	-.15	-.03	1.00	-.16	.07	-.28	-.45	-.35	-.35
X7	.16	.12	.04	-.22	-.21	-.16	1.00	.28	.36	.45	.54	.54
X8	.01	.07	.35	-.29	-.16	.07	.28	1.00	.27	.45	.51	.51
I2	.91	.91	.38	.01	-.57	-.28	.36	.27	1.00	.70	.91	.92
IM	.45	.47	.45	.45	-.48	-.45	.45	.45	.70	1.00	.68	.69
I1	.73	.73	.47	-.27	-.43	-.35	.54	.51	.91	.68	1.00	1.00
IE	.74	.74	.47	-.25	-.44	-.35	.54	.51	.92	.69	1.00	1.00

Off-Diagonal Entries in the Red are Significant at 5% Probability

Fig.-IV. Relationship Among Different Composite Indices [Ref. Indices Table-IV.B]



Note: FORTRAN codes for computing inclusive and egalitarian indices may be obtained on request to mishrasknehu@yahoo.com